Basic topology 4

Graphically analyze the convexity of the following sets.

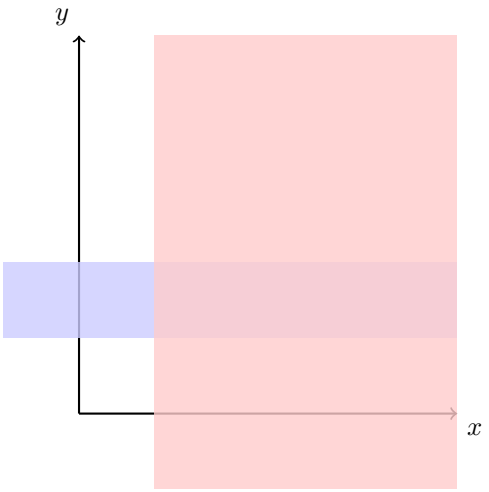
1. $A = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq y \leq 2 \text{ and } x \geq 1\}$
2. $D = \{(x, y) \in \mathbb{R}^2 \mid y \leq \frac{1}{x}\}$

Solutions

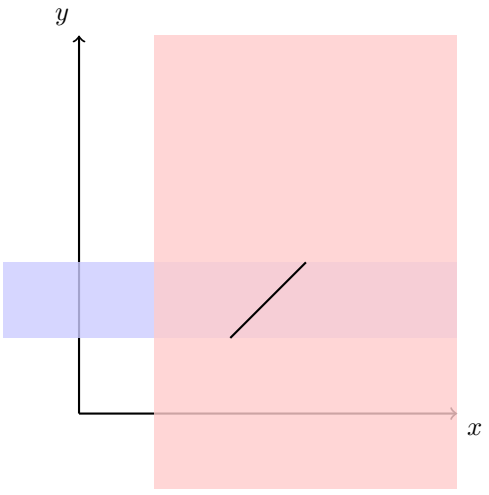
1. A set is convex if, given any two points within the set, the segment that joins them is contained within the set.

   Formally, given a set A we say that it is convex if for every $x, y \in A$ and for every $\lambda \in \mathbb{R}$ it holds that $\lambda x + (1 - \lambda)y \in A$. (Here, by assigning different values to λ we get all the points that are in the segment that joins x and y)

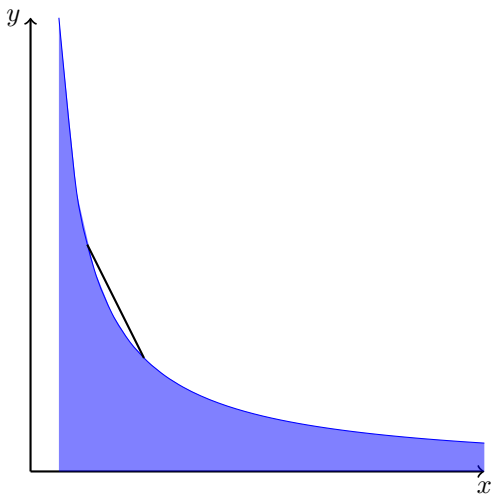
   Let's see the graph of the set A:



   The set A consists of points that are painted blue and red simultaneously. We can observe that, given any two points from A, the segment that joins them is included in A. **Therefore, it is convex.** See for example.



2. Let's see the graph of the set D for the first quadrant



This set is not convex; for example, the segment that joins the points $(0.5, 2)$ and $(1, 1)$ is not contained in D.